Toward a systematic holographic QCD: a braneless approach

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ABSTRACT: Recently a holographic model of hadrons motivated by AdS/CFT has been proposed to fit the low energy data of mesons. We point out that the infrared physics can be developed in a more systematic manner by exploiting backreaction of the nonperturbative condensates. We show that these condensates can naturally provide the IR cutoff corresponding to confinement, thus removing some of the ambiguities from the original formulation of the model. We also show how asymptotic freedom can be incorporated into the theory, and the substantial effect it has on the glueball spectrum and gluon condensate of the theory. A simple reinterpretation of the holographic scale results in a non-perturbative running for α_s which remains finite for all energies. We also find the leading effects of adding the higher condensate into the theory. The difficulties for such models to reproduce the proper Regge physics lead us to speculate about extensions of our model incorporating tachyon condensation.

KEYWORDS: QCD, AdS-CFT Correspondence, Field Theories in Higher Dimensions, Confinement.

Contents

1. Introduction

QCD is a perennially problematic theory. Despite its decades of experimental support, the detailed low-energy physics remains beyond our calculational reach. The lattice provides a technique for answering nonperturbative questions, but to date there remain open questions that have not been answered. For instance, the low-energy scalar spectrum is a puzzle. There are a lot of experimentally observed states, however their composition (glueball vs. quarkonium) and their mixings are not well understood. The difficulty for any theory trying to make progress in this direction is to understand the interaction between the scalar states and the vacuum condensates of QCD. In this paper we attempt to incorporate the effects of the vacuum condensates into the holographic model of QCD as a first step toward understanding the scalar sector in the context of these models.

The SVZ sum rules [1] are a powerful theoretical tool for relating theoretically solid facts about perturbative QCD with experimental data in the low-energy region. The basic observation is that the correlator for a current J

$$\Pi(q) = i \int \mathrm{d}^4 x \; e^{iqx} \langle 0 | T J(x) J(0) | 0 \rangle \tag{1.1}$$

may be expanded in a Wilson OPE that is valid up to some power (where instanton corrections begin to invalidate the local expansion), $(1/Q^2)(\mathrm{d}\Pi/\mathrm{d}Q^2) = \sum C_{2d} \langle \mathcal{O}_{2d} \rangle$, where the $\mathcal{O}_{2d}$ are gauge-invariant operators of dimension $2d$. In the deep Euclidean domain the coefficients C_{2d} are calculable. On the other hand, the correlator relates to observable quantities; for instance if J is the vector current then $\mathrm{Im}\Pi(s)$ is directly proportional to the spectral density $\rho(s)$, measurable from the total cross section $\sigma(e^+e^- \to \text{hadrons})$. The SVZ sum rules, then, relate Wilson OPE expansions to measurable quantities. They can become useful for understanding detailed properties of the first resonances when one takes a Borel transform that suppresses the effect of higher resonances. As it turns out, keeping only low orders of perturbation theory in the coefficients C_{2d}, one still obtains reasonable agreement with data, so the assumption that the largest corrections arise from the condensates is a good one.

Recently another technique for understanding the properties of low-lying mesons has arisen in the form of AdS/QCD. The phenomenological model constructed on this basis [2] takes as its starting points the OPE (much as in the SVZ sum rules) and the AdS/CFT correspondence [3]. The idea is straightforward: rather than attempting to deform the usual Type IIB on $AdS_5 \times S^5$ to obtain a theory more like QCD, one starts with QCD and attempts to build a holographic dual. Of course in detail such a program is bound to eventually run up against difficulties from α' corrections, g_s corrections, the geometry of the five compact dimensions (or the proper definition of a noncritical string theory) and other issues. However, one can set aside these problems, begin with a relatively small set of fields needed to model the low-lying states in QCD, and see how well the approximation works. In this phenomenological approach with bulk fields placed in the Randall-Sundrum background [4], and the AdS space cut off with a brane at a fixed $z = z_c$ in the infrared a surprisingly good agreement with the physics of the pions, ρ, and a_1 mesons has been found [2] (for more recent work on AdS/QCD see [5, 6]). However, there are several obvious limitations to this model. For example, it does not take into account the leading power corrections to the OPE in the UV (the effects of the vacuum condensates), or the corrections coming from logarithmic running. It includes power corrections only where they are needed to reproduce chiral symmetry breaking. Also, the theory has a single mass scale (set by the location of IR cutoff brane) which determines the mass scales in all the different sectors in QCD.

Here we show how the effect of the vacuum condensates and of asymptotic freedom can be simply incorporated into the model. We will limit ourselves to pure Yang-Mills theory without quarks, though we expect that most of the aspects of incorporating quarks should be relatively straightforward. For the vacuum condensates one has to introduce a dynamical scalar with appropriate mass term and potential coupled to gravity. There will

be a separate field for every gauge invariant operator of QCD, and a non-zero condensate will lead to a non-trivial profile of the scalar in the bulk. While the effects of these condensates on the background close to the UV boundary are small (though not always negligible), they will become the dominant source in the IR and effectively shut off the space in a singularity (and without having to cut the space off by hand). This resolves several ambiguities in choosing the boundary conditions at the IR brane, and the various condensates will also be able to set different mass scales in the different sectors of QCD. Asymptotic freedom can be achieved by properly choosing the potential for the scalar corresponding to the Yang-Mills gauge coupling. Incorporating asymptotic freedom will have an important effect on the glueball sector since a massless Goldstone field will pick up a mass from the anomalous breaking of scale invariance.

It is natural to ask why we should expect QCD to have any useful holographic dual. The most well understood examples of holography are in the limit of large N and $g_{\mathrm{YM}}^2 N \gg 1$, far from the regime of real-world QCD, which apparently would be a completely intractable string theory with a large value of α'. On the other hand, the large N approximation has frequently been applied in QCD phenomenology. A major concern in applying holography to QCD is whether the dual should be local, or whether it can have higher-derivative α' corrections. The α' corrections are associated with the massive stringy excitations in the bulk, which once integrated out yield complicated Lagrangians for the remaining light fields. However, we can think of this physics in a different way. Holography is closely related to the renormalization group [7]; the coordinate z can be identified with μ^{-1}, where μ is the renormalization group scale. AdS/CFT identifies massive bulk fields with higher-dimension operators in the field theory. From this point of view, α' corrections to the physics of light bulk fields are associated with the effects of higher-dimension operators coupled to the low-dimension operators in the RG flow. The OPE tells us that, at large Q^2, the effects of these higher-dimension operators in the field theory are controllably small. From this point of view, despite the apparently large α' corrections, it is reasonable to begin with a local bulk action in terms of fields corresponding to the low dimension operators of QCD. The example of SVZ gives us hope that this can correctly capture physics of light mesons. For highly excited mesons, which in QCD look like extended flux tubes, and thus feel long distances, it is more probable that the strong IR physics will mix different operators and that our neglect of α'-like corrections will become more troublesome. It is a general problem, in fact, that physics of highly excited hadrons is troublesome in these models [8]. Recent proposals for backgrounds with correct Regge physics [9] offer some hope of addressing this problem. We will offer some further comments on how a closed-string tachyon might give a dynamical explanation of such a background, but there is clearly much more to be done along these lines.

The paper is organized as follows. In section 2 we will remind the reader of the basic formulation of AdS/QCD on Randall-Sundrum backgrounds as in ref. [2], and discuss some of its shortcomings. In section 3 we will discuss a class of models that do not incorporate the running of the QCD coupling, but do incorporate the effects of the lowest vacuum condensate. We will point out that these backgrounds too have some shortcomings, but improve on the RS backgrounds, and may provide a useful setting for exploring some

questions. We calculate the gluon condensate and point out that there is a zero mode in the glueball spectrum due to the spontaneous breaking of scale invariance. In section 4 we discuss the construction of 5D theories with asymptotic freedom. We calculate the gluon condensate and provide a first estimate of masses for the 0^{++} glueballs from these backgrounds. We point out a possible interpretation of the running coupling where α_s remains finite for all energies. In section 5 we show how to systematically incorporate the effects of the higher condensates. We give a background including the effect of $\langle \mathrm{Tr F}^3 \rangle$ and show how it affects the gluon condensate and the glueball spectrum. In section 6 we discuss the difficulties of reproducing the correct Regge physics, and speculate that closed string tachyon dynamics could perhaps be responsible for reproducing the necessary IR background. Finally we provide the conclusions and some outlook in section 7.

2. AdS/QCD on Randall-Sundrum backgrounds

We briefly review the AdS/QCD model on Randall-Sundrum backgrounds [2]. One assumes that the metric is exactly AdS in a finite region $z = 0$ to $z = z_c$, i.e.

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^\mu dx^\nu \eta_{\mu\nu} - dz^2\right), \ 0 \leq z \leq z_c, \tag{2.1}$$

where $z_c \sim \Lambda_{\mathrm{QCD}}^{-1}$ determines the scale of the mass spectrum. It is assumed that there is a brane (the "infrared brane") at $z = z_c$; in practice one assumes a UV boundary at $z = \epsilon$ and sends $\epsilon \to 0$ at the end of calculations. One puts a field $\phi_{\mathcal{O}}$ in the bulk for every gauge invariant operator $\mathcal{O}$ in the gauge theory. If $\mathcal{O}$ is a p-form of dimension Δ, ϕ has a 5D mass $m_5^2 = (\Delta - p)(\Delta + p - 4)$. This operator has UV boundary conditions dictated by the usual AdS/CFT correspondence. On the other hand, IR boundary conditions are less clear, and one in principle can add localized terms on the IR brane.

For instance, in the original papers the treatment involves the rho and a_1 mesons and the pions. There are bulk gauge fields, A_M^L and A_M^R (where M is a 5D Lorentz index) coupling to the operators $\bar{q}_L \gamma^\mu t^a q_L$ and $\bar{q}_R \gamma^\mu t^a q_R$. There is also a bulk scalar $X^{\alpha\beta}$ coupling to the operator $\bar{q}_R^\alpha q_L^\beta$, where α and β are flavor indices. The scalar $X^{\alpha\beta}$ is *assumed* to have a profile proportional to $\delta^{\alpha\beta} \left(m_\alpha z + \langle q_\alpha \bar{q}_\alpha \rangle z^3\right)$, based on the Klebanov-Witten result [11] that a nonnormalizable term in a scalar profile corresponds to a perturbation of the Lagrangian by a relevant operator (in this case, $m\bar{q}q$), while a normalizable term corresponds to a spontaneously generated VEV for the corresponding field theory operator. The profile for $X(z)$ couples differently to the vector and axial vector mesons and achieves the $\rho - a_1$ mass splitting. Furthermore, the pions arise as pseudo-Nambu-Goldstone modes of broken chiral symmetry, and the Gell-Mann-Oakes-Renner relationship is satisfied. Several other constants in the chiral Lagrangian are determined to around 10% accuracy.

There are several limitations to this model. One is that it does not take into account the leading power corrections to the OPE in the UV, or the corrections coming from logarithmic running. (The scalar X representing $\langle q\bar{q} \rangle$ reproduces some of the power corrections distinguishing those operators sensitive to chiral symmetry breaking from those that are not, but power corrections unrelated to chiral symmetry breaking were omitted.) The SVZ

sum rules work fairly well with just the leading power correction taken into account [1], and these power corrections can be incorporated into the form of the metric with a simple ansatz [6]. However, backreaction has not been taken into account in such studies, so the Einstein equations will not be exactly satisfied. Also, in the SVZ sum rule approach, leading corrections to the OPE essentially determine the mass scale of the lightest resonance in the corresponding channel. In the Randall-Sundrum approach, the leading correction to the OPE and the IR wall at z_c *both* influence the corresponding mass scale. This has effects on the spectrum.

For instance, in QCD the mass scales associated with mesons made from quarks and mesons made from gluons are very different [10]. One can see this in the OPE. For instance, in the tensor 2^{++} channel, the leading corrections to the OPE come from the same operator in the $q\bar{q}$ case as in the G^2 case, but the coefficients are very different. In the Randall-Sundrum approach, without turning on a background field VEV the mass scale for both of these channels is set by z_c^{-1}. One can turn on a background VEV and give the fields different couplings to it to reproduce the difference in the OPEs, but z_c^{-1} will continue to play a role in setting their masses.

Another limitation is that these backgrounds lack asymptotic freedom. If one defines the theory with a cutoff at which α_s has some finite value, this might not appear to be of central importance. On the other hand, we understand that in real QCD the values of the condensates are determined by the QCD scale at which the perturbative running coupling blows up, so achieving asymptotic freedom can allow such a relationship and make the model less *ad hoc*. Furthermore, we will see that the lack of proper conformal symmetry breaking can lead to a massless scalar glueball state (i.e., the model has a "radion problem"), which is most satisfactorily resolved by incorporating asymptotic freedom.

In summary, the AdS/QCD models on hard-wall backgrounds work surprisingly well for some quantities, but have obvious drawbacks. There are ambiguities in IR boundary conditions, and the existence of a single IR wall influencing all fields obscures the relationship between masses of light resonances and power corrections discovered by Shifman, Vainshtein, and Zakharov. Luckily, there is a simple remedy to these difficulties: we simply remove the IR brane, and allow the growth of the condensates to dynamically cut the space off in the infrared.

3. Vacuum condensates as IR cutoff

To model the pure gauge theory we begin with the action for five-dimensional gravity coupled to a dilaton (in the Einstein frame):

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5 x \sqrt{g}\left(-\mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \right). \tag{3.1}$$

Here κ^2 is the 5 dimensional Newton constant and R is the AdS curvature (related to the the (negative) bulk cosmological constant as $R^{-2} = -\frac{\kappa^2}{6}\Lambda$. Note, that ϕ is dimensionless here. The dilaton will couple to the gluon operator $G_{\mu\nu}G^{\mu\nu}$. The fact that there is a non-vanishing gluon condensate in QCD is expressed by the fact that the dilaton will have

a non-trivial background. We can find the most general such background by solving the coupled system of the dilaton equation of motion and the Einstein's equation, under the ansatz that we preserve four-dimensional Lorentz invariance while the fifth dimension has a warp factor:

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$
$$\phi = \phi(y). \tag{3.2}$$

The coupled equations for $A(y), \phi(y)$ will then be

$$4A'^2 - A'' = 4R^2$$
$$A'^2 = \frac{\phi'^2}{24} + \frac{1}{R^2}$$
$$\phi'' = 4A'\phi'. \tag{3.3}$$

A simple way of solving these equations is to use the superpotential method [12, 13], that is define the function $W(\phi)$ such that

$$A'(y) = W(\phi(y))$$
$$\phi'(y) = 6\frac{\partial W}{\partial \phi}. \tag{3.4}$$

This is always possible, and the equation determining the superpotential is given by

$$V = 18\left(\frac{\partial W}{\partial \phi}\right)^2 - 12W^2 = -\frac{12}{R^2}. \tag{3.5}$$

To solve for the most general superpotential consistent with our chosen (constant) potential, it is useful to parameterize it in terms of a "prepotential" w as [13]

$$W = \frac{1}{R}\left(w + \frac{1}{w}\right)$$
$$W' = \sqrt{\frac{2}{3}}\frac{1}{R}\left(w - \frac{1}{w}\right), \tag{3.6}$$

which is chosen such that (3.5) is automatically satisfied. The consistency condition of the two equations in (3.6) implies a simple equation for the prepotential:

$$w' = \sqrt{\frac{2}{3}}w, \tag{3.7}$$

and thus for the superpotential we find

$$W(\phi) = \frac{1}{R}\left(Ce^{\sqrt{\frac{2}{3}}\phi} + C^{-1}e^{-\sqrt{\frac{2}{3}}\phi}\right). \tag{3.8}$$

With the superpotential uniquely determined (up to a constant C) we can then go ahead and integrate the equations in (3.4). The result is

$$\phi(y) = \sqrt{\frac{3}{2}}\log\left[C\tanh 2(y_0 - y)/R\right],$$
$$A(y) = -\frac{1}{4}\log\left[\cosh 2(y_0 - y)/R\sinh 2(y_0 - y)/R\right] + A_0, \tag{3.9}$$

where y_0 and A_0 are integration constants. These are the solutions also found in [13, 14]. In order to have $A(y)$ asymptotically equal to y (for large negative y), we will fix $A_0 = \frac{y_0}{R} - \frac{1}{2}\log 2$. Note, that there is another branch of the solution where in which tanh is replaced by coth in the solution for ϕ. In order to have a form of the solution that is more familiar and useful, we make a coordinate transformation $e^{\frac{y-y_0}{R}} = z/z_c$. This will recast the solution in a form that matches the usual conformal coordinates x^μ, z near $z = 0$:

$$\mathrm{d}s^2 = \left(\frac{R}{z}\right)^2 \left(\sqrt{1 - \left(\frac{z}{z_c}\right)^8}\, \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}z^2\right) \tag{3.10}$$

$$\phi(z) = \sqrt{\frac{3}{2}}\log\left(\frac{1 + \left(\frac{z}{z_c}\right)^4}{1 - \left(\frac{z}{z_c}\right)^4}\right) + \phi_0. \tag{3.11}$$

Here z_c is a new parameter determining the IR scale, and we expect $z_c \sim \Lambda_{\mathrm{QCD}}^{-1}$. The point $z = z_c$ is a naked singularity, which we must imagine is resolved in the full string theory. Also note that the first correction to the AdS_5 metric goes as z^8.

3.1 Gluon condensate

We have seen that the metric incorporates power corrections to the pure AdS solution, which we want to identify with the effects of the gluon condensate. Below we would like to make this statement more precise. According to the general rules of the AdS/CFT correspondence given some field $\varphi_{\mathcal{O}}$ on this background representing an operator $\mathcal{O}$ in QCD, the deep-Euclidean correlator of $\mathcal{O}$ will have a Q^{-8} correction if $\varphi_{\mathcal{O}}$ has no dilaton coupling, and a Q^{-4} correction if $\varphi_{\mathcal{O}}$ couples to ϕ. Note that near $z = 0$, the dilaton behaves $\phi_0 + \sqrt{6}\frac{z^4}{z_c^4}$. This is in agreement with the expectation that a field coupling to an operator with dimension Δ has two solutions, $z^{\Delta-d}$ and z^Δ. In our case $\Delta = d = 4$, and we expect the constant piece to correspond to the source for the operator $\mathrm{Tr}G^2$ and coefficient of the z^4 to give the gluon condensate. The precise statement [11] from AdS/CFT is that if solution to the classical equations of motion Φ has the form near the boundary

$$\Phi(x, z) \to z^{d-\Delta}[\Phi_0(x) + \mathcal{O}(z^2)] + z^\Delta[A(x) + \mathcal{O}(z^2)] \tag{3.12}$$

then the condensate (one-point function) of the operator $\mathcal{O}$ is given by

$$\langle\mathcal{O}(x)\rangle = (2\Delta - d)A(x). \tag{3.13}$$

However, to apply this to the solution in (3.11) we need to make sure that the appropriate normalization of the fields is used. The expression (3.13) is derived assuming a scalar field action of the form $1/2\int d^4x dz/z^3(\partial_M\phi)^2$. Comparing this with the action used here (3.1) we find

$$\langle\mathrm{Tr}G^2\rangle = 4\sqrt{3}\sqrt{\frac{R^3}{\kappa^2}}\frac{1}{z_c^4}. \tag{3.14}$$

In order to be able to relate this expression for the condensate we need to find an expression for R^3/κ^2. This can be done by requiring that the leading term in the OPE of

the gluon operator G^2 is correctly reproduced in the holographic theory. One can simply calculate the leading term in the OPE in QCD [15]

$$\int d^4x \langle G^2(x) G^2(0) \rangle e^{iqx} = -\frac{(N^2-1)}{4\pi^2} q^4 \log \frac{q^2}{\mu^2} + \dots \tag{3.15}$$

The same quantity can be calculated in the gravity theory by evaluating the action for the scalar field with a give source $\phi_0(q)$ in the UV. The general expression for the action is obtained (after integrating by parts and using the bulk equation of motion)

$$S_{5D} = \frac{1}{2\kappa^2} \int d^4x \frac{R^3}{z^3} \frac{1}{2} \phi \partial_z \phi |_{z \to 0}. \tag{3.16}$$

In order to find the action one can use the bulk equation of motion close to the UV for the scalar field given by

$$\phi'' + q^2 \phi - \frac{3}{z} \phi' = 0. \tag{3.17}$$

Requiring that the wave function approaches 1 around $z = 0$ will fix the leading terms in the wave function:

$$\phi(z) = 1 - \frac{1}{32} q^4 z^4 \log q^2 z^2 + \frac{1}{4} q^2 z^2 + \dots. \tag{3.18}$$

Taking the second derivative of the action we find

$$\int d^4x \langle G^2(x) G^2(0) \rangle e^{iqx} = -\frac{R^3}{16\kappa^2} q^4 \log q^2/\mu^2 + \dots, \tag{3.19}$$

from which we get the identification

$$\frac{R^3}{\kappa^2} = \frac{4(N^2-1)}{\pi^2}. \tag{3.20}$$

Using this result we get a prediction for the gluon condensate

$$\langle \mathrm{Tr} G^2 \rangle = \frac{8}{\pi z_c^4} \sqrt{3(N^2-1)}. \tag{3.21}$$

3.2 The glueball spectrum

Now we wish to solve for the scalar glueball spectrum, which is associated with scalar fluctuations about the dilaton-metric background. The analogous calculation in the supergravity model has been performed in [16]. We should solve the coupled radion-dilaton equations, as often there is a light mode from the radion. In other words, we should solve for eigenmodes of the coupled Einstein-scalar system. This has been worked out in detail for a generic scalar background in [17]: the linearized metric and scalar ansatz is given by:

$$ds^2 = e^{-2A(y)}(1 - 2F(x,y))dx^\mu dx^\nu \eta_{\mu\nu} - (1 + 4F(x,y))dy^2$$

$$\phi(x,y) = \phi_0(y) + \frac{3}{\kappa^2 \phi_0} \left(F'(x,y) - 2A'(y)F(x,y) \right). \tag{3.22}$$

This will satisfy the coupled Einstein-scalar equations if $F = F(y)e^{iq \cdot x}$ with $q^2 = m^2$ and $F(y)$ satisfies the differential equation

$$F'' - 2A'F' - 4A''F - 2\frac{\phi_0''}{\phi_0'}F' + 4A'\frac{\phi_0''}{\phi_0'}F = -e^{-2A}m^2 F. \tag{3.23}$$

Using the solutions for $A(y)$ and $\phi_0(y)$ from eq. 3.9, and an ansatz $F(x, y) = F(y)e^{iq \cdot x}$, with $q^2 = -m^2$, this becomes:

$$F''(y) + \frac{10}{R} \coth \frac{4(y - y_0)}{R} F'(y) + \left(\frac{16}{R^2} + m^2 \frac{e^{2y_0/R}}{\sqrt{2 \sinh \frac{4(y_0 - y)}{R}}} \right) F(y) = 0. \qquad (3.24)$$

Demanding a normalizable solution, we need that (in the z coordinates) $\int dz \sqrt{g} |\varphi(z)|^2$ and $\int dz \sqrt{g} g^{55} |\partial_z \varphi(z)|^2$ be finite. Thus we need $\varphi(z) \sim z^4$ at small z. We need to be somewhat more careful about solving for F: in the z coordinates, we have that

$$z \to 0 : z \frac{dF}{dz} - 2z \frac{dA}{dz} F \to z \frac{dF}{dz} - 2F \sim \varphi(z), \qquad (3.25)$$

so that $F(z) \sim z^2$ near $z = 0$ is compatible with our assumptions on the behavior of $\varphi(z)$. As usual, this equation can be solved using the shooting method: the differential equation is solved numerically starting from the UV boundary with arbitrary normalization the BC following from (3.25) (the numerics is very insensitive to the choice of the actual UV BC for the higher modes) for varying values of m^2. For discrete values of m^2 the wave function will be normalizable (which numerically is equivalent to requiring a Neumann BC at the location of the singularity). This way we find (in units of z_c^{-1}) glueballs with masses 6.61, 9.84, 12.94, and 15.98 (and so forth, with regular spacing in mass).

There is also a serious problem: we find a massless mode for the radion (with the $F(z) \sim z^2$ UV boundary conditions). This can be understood as follows: in real QCD, the classical conformal symmetry is broken by the scale anomaly. However, in our model, it is broken by the z^4 profile of the dilaton. AdS/CFT tells us we should understand turning on such a normalizable background in the UV as a spontaneous symmetry breaking. In Randall-Sundrum models, one similarly has a radion problem and needs to invoke (for instance) a Goldberger-Wise stabilization [18] to avoid a massless mode. The radion has not been part of previous investigations of glueballs on Randall-Sundrum backgrounds [19], so such studies have essentially assumed that the stabilization mechanism removes a light mode from the spectrum. However, an added Goldberger-Wise field does not seem to correspond to an operator of QCD, so it goes against the spirit of the AdS/CFT correspondence. It is apparent that a palatable solution of the radion problem in AdS/QCD demands a 5D treatment of scale dependence that mirrors that in 4D. This motivates us to search for backgrounds incorporating asymptotic freedom, which also allows us to begin approaching a more detailed matching to perturbative QCD.

4. Incorporating asymptotic freedom

We have shown in the previous section a background that incorporates the lowest QCD condensate $\text{Tr}F^2$ and which automatically provides an IR cutoff via the backreaction of the metric. However, this setup is certainly too simplistic even to just produce the main features of QCD: for example asymptotic freedom is not reproduced in that setup. It is the dilaton field that also sets the QCD coupling constant, and by approaching a constant value

the model in the previous section actually describes a theory that approaches a conformal fixed point in the UV, rather than QCD. One may think that this is not an important difference for the IR physics, but this is not quite right. For example as we have seen it introduces a "radion problem."

Thus we set out to find a potential for the dilaton that will reproduce the logarithmic running of the coupling. We assume that similarly to string theory the gauge coupling is actually given by $e^{b\phi(z)}$ (where b is a numerical constant). We will find a result consistent with expectations from string theory.[1]

We assume that our action is:

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^5 x \sqrt{g} (-\mathcal{R} - V(\phi) + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi), \qquad (4.1)$$

where now we will try to determine $V(\phi)$ such that we reproduce asymptotic freedom. If we require that the coupling runs logarithmically, and as usual identify the energy scale with the inverse of the AdS coordinate z we need to have a solution of the form

$$e^{b\phi(z)} = \frac{1}{\log \frac{z_0}{z}}, \qquad (4.2)$$

where $z_0 = \Lambda_{\mathrm{QCD}}^{-1}$ and we do not fix b a priori. Then going to the y coordinates as usual via the definition $e^{y/R} = z/R$, we will find

$$e^{b\phi(y)} = \frac{R}{y_0 - y}. \qquad (4.3)$$

If we now assume that this solution follows from a superpotential W then $\phi'(y) = 6\frac{\partial W}{\partial \phi} = \frac{1}{b(y_0-y)} = \frac{1}{bR} e^{b\phi}$. This implies that $W(\phi) = \frac{1}{6Rb^2} e^{b\phi} + W_0$. Now that we have found the form of the superpotential, we can easily solve for the warp factor: $A'(y) = W(\phi(y)) = \frac{1}{6b^2} \frac{1}{(y-y_0)} + W_0$, and hence $A(y) = A_0 + W_0 y + \frac{1}{6b^2} \log \frac{R}{y_0-y}$. In z coordinates, this becomes:

$$A(z) = A_0 + W_0 R \log z/R - \frac{1}{6b^2} \log \log z_0/z, \qquad (4.4)$$

and hence $e^{-2A(z)} = e^{-2A_0} (R/z)^{2W_0 R} (\log z_0/z)^{1/(3b^2)}$. From this we conclude that we should take $A_0 = 0$ and $W_0 = \frac{1}{R}$ to get a solution that looks AdS-like up to some powers of $\log z_0/z$.

The potential corresponding to this superpotential is then given by

$$V(\phi) = 18 \left(\frac{\partial W}{\partial \phi}\right)^2 - 12W^2 = -\frac{1}{3b^2 R^2} \left(\left(\frac{1}{b^2} - \frac{3}{2}\right) e^{2b\phi} + 12 e^{b\phi} + 36b^2 \right). \qquad (4.5)$$

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[1]Other discussions of backgrounds with logarithmically running coupling can be found in [20, 21].

This is *particularly* simple in the case that $b = \pm\sqrt{\frac{2}{3}}$. In that case we have simply

$$V(\phi) = -\frac{6}{R^2}e^{\pm\sqrt{\frac{2}{3}}\phi} - \frac{12}{R^2} \tag{4.6}$$

$$W(\phi) = \frac{1}{R}\left(\frac{1}{4}e^{\pm\sqrt{\frac{2}{3}}\phi} + 1\right) \tag{4.7}$$

$$\phi = \mp\sqrt{\frac{3}{2}}\log\frac{y_0 - y}{R} = \mp\sqrt{\frac{3}{2}}\log\log\frac{z_0}{z} \tag{4.8}$$

$$A = \frac{y}{R} + \frac{1}{4}\log\frac{R}{y_0 - y} = \log\frac{z}{R} - \frac{1}{4}\log\log\frac{z_0}{z}, \tag{4.9}$$

and thus the metric will be

$$ds^2 = \left(\frac{R}{z}\right)^2\left(\sqrt{\log\frac{z_0}{z}}dx^\mu dx^\nu\eta_{\mu\nu} - dz^2\right) = \frac{e^{-2\frac{y}{R}}}{\sqrt{y_0 - y}}\,dx^\mu dx^\nu\eta_{\mu\nu} - dy^2. \tag{4.10}$$

Our dilaton in (4.1) is normalized in an unusual way, nevertheless (4.6) is recognizable as a potential that commonly occurs in nonsupersymmetric string theory backgrounds: a cosmological constant plus a term exponential in the dilaton. This is quite reasonable from the string theory perspective, where such a term can arise from dilaton tadpoles in critical backgrounds or from the central charge in noncritical backgrounds. In fact, the factor $\sqrt{2/3}$ arises from string theory considerations in a simple way, which may be an amusing coincidence or may have more significance. Suppose that there is a noncritical string theory in 5 dimensions. Its action in string frame has the form [22]

$$S = \frac{1}{2\kappa_0^2}\int d^5x(-G)^{1/2}e^{-2\Phi}\left(C + R + 4\partial_\mu\Phi\partial^\mu\Phi + \cdots\right), \tag{4.11}$$

where C is proportional to the central charge and is nonvanishing since we are dealing with a noncritical string. Now we go to Einstein frame:

$$S = \frac{1}{2\kappa^2}\int d^5x(-\tilde{G})^{1/2}\left(Ce^{4\tilde{\Phi}/3} + \tilde{R} - \frac{4}{3}\partial_\mu\tilde{\Phi}\tilde{\partial}^\mu\tilde{\Phi} + \cdots\right), \tag{4.12}$$

and finally we note that comparing to our normalization above, $\tilde{\Phi} = \sqrt{3/8}\phi$, so that $e^{4\tilde{\Phi}/3} = e^{\sqrt{2/3}\phi}$.

4.1 The glueball spectrum

To calculate the glueball spectrum we can apply eq. (3.23) for the background in (4.8), (4.9). This equation (transformed to z coordinates) reduces to (in units of R)

$$z^2F''(z) - z\left(1 + \frac{5}{2\log\frac{z_0}{z}}\right)F'(z) + \left(\frac{4}{\log\frac{z_0}{z}} + \frac{m^2z^2}{\sqrt{\log\frac{z_0}{z}}}\right)F(z) = 0. \tag{4.13}$$

Using the shooting method again we find (in units of z_0^{-1}) glueballs at 2.52, 5.45, 8.16, and 10.81. In particular this background seems to have a light mode from the radion, but not any zero mode.

Lattice estimates put the first 0^{++} glueball in pure SU(3) gauge theory at approximately 1730 MeV, and the second at about 2670 MeV, with uncertainties of order 100 MeV [23]. Thus they put the ratio of the first and second scalar glueball masses about about 1.54, whereas we find a significantly larger value of 2.16. While the lattice errors are still fairly large, this probably indicates that we are not so successful at precisely determining properties of the second scalar glueball resonance. Since we undoubtedly fail to properly describe highly excited resonances, this is not so surprising. If we set z_0^{-1} to match the lattice estimate for the first glueball mass, we find

$$z_0^{-1} \approx 680 \text{ MeV.} \tag{4.14}$$

One can also calculate the spectrum of spin 2^{++} glueball masses by solving the fluctuations of the Einstein equation around the background. The resulting differential equation we find is

$$z \log \frac{z_0}{z} f''(z) - \left(1 + 3 \log \frac{z_0}{z}\right) f'(z) + m^2 z \sqrt{\log \frac{z_0}{z}} f(z) = 0. \tag{4.15}$$

Using the shooting method (and imposing Dirichlet BC on the UV boundary) we find the lightest modes at 4.03, 6.56, ... in units of $1/z_0$. It is important to point out that the lowest spin 2^{++} glueball is naturally heavier in this setup than the spin 0^{++} glueball due to the mixing of the radion with the dilaton. In the usual supergravity solutions the spin 0^{++} and 2^{++} glueballs usually end up degenenerate (in contradiction to lattice simulations). This is for example the case in the AdS black hole solution of Witten analyzed in [16] (the additional light scalar modes identified in [24] do not correspond to QCD modes).

4.2 Power corrections and gluon condensate

We would now like to evaluate the gluon condensate in this theory assuming that the the IR scale z_0 is fixed by the value of the lightest glueball mass. As usual one needs to calculate the 5D action corresponding to a fixed source term turned on for the QCD coupling and to get the condensate (one -point function) we need to differentiate the 5D action with respect to the source. Ordinarily, the computation of the 5D action on a given solution in AdS/QCD reduces to simply a boundary term. However, in our case it is not so simple: our potential is not just a mass term, so that the bulk piece $V - \frac{\phi}{2} \frac{\partial V}{\partial \phi}$ is not set to zero by the equations of motion. As a result, the action also has a "bulk" piece.

We evaluate the 5D action as a function of z_0, imposing a UV cutoff at ϵ. The action is given by:

$$\frac{1}{2\kappa^2} \int_{z=\epsilon}^{z_0} \left(\frac{R}{z}\right)^5 \log \frac{z_0}{z} \left(-\mathcal{R} - \frac{1}{2} z^2 \phi'(z)^2 + \frac{12}{R^2} + \frac{6}{R^2} e^{\sqrt{\frac{2}{3}}\phi(z)}\right) dz. \tag{4.16}$$

This integral can be performed explicitly:

$$\frac{1}{2\kappa^2} \left[\frac{1}{2z^4} + \frac{2 \log \frac{z_0}{z}}{z^4}\right]_{z=\epsilon}^{z_0} \tag{4.17}$$

We drop the UV divergent terms (the $1/\epsilon^4$ pieces) assuming that there will be counter terms absorbing these. Then the explicit expression for the action will be

$$S(z_0) = \frac{1}{4\kappa^2 z_0^4}. \qquad (4.18)$$

Note that this is more easily calculated as

$$S(z_0) = \frac{1}{2\kappa^2} \int d^4x \, 2\sqrt{g_{4D}} \, W(\phi), \qquad (4.19)$$

evaluated at the boundary $z = z_0$, where g_{4D} is the induced 4D metric at the boundary. (This observation has been made before in ref. [25].) To see this, we use the following relations:

$$-\mathcal{R}(y) = -20A'(y)^2 + 8A''(y) = -20W(\phi)^2 + 48\left(\frac{\partial W}{\partial \phi}\right)^2 \qquad (4.20)$$

$$-\frac{1}{2}\phi'(y)^2 = -18\left(\frac{\partial W}{\partial \phi}\right)^2 \qquad (4.21)$$

$$-V(\phi) = -18\left(\frac{\partial W}{\partial \phi}\right)^2 + 12W^2 \qquad (4.22)$$

to see that $\sqrt{g}S_{5D}$, evaluated on the solution, is

$$e^{-4A(y)}\left(-8W^2 + 12\left(\frac{\partial W}{\partial \phi}\right)^2\right) = 2\frac{d}{dy}\left(e^{-4A(y)}W(\phi(y))\right). \qquad (4.23)$$

This makes it clear that we can use the superpotential as a counterterm on the UV boundary to cancel the terms diverging as $\epsilon \to 0$ (which we dropped above.)

It turns out that $\frac{1}{\kappa^2}$ is almost precisely as in the background without asymptotic freedom (because, for the fluctuating modes, corrections to the wavefunction near $z = 0$ are small α_s corrections), so one can still use (3.20) to find the value of R^3/κ^2. However, there is a slight subtlety: we found a value for $\frac{1}{\kappa^2}$ assuming a source coupled to $\mathrm{Tr}G^2$. In fact in our case we have fixed the numerical factor in the correspondence of $e^{\sqrt{2/3}\phi}$ based on its asymptotic behavior. Using the expression for the coupling in a pure YM theory

$$\alpha_{\mathrm{YM}}(Q) = \frac{2\pi}{\frac{11}{3}N_c \log \frac{Q}{\Lambda_{\mathrm{QCD}}}} \qquad (4.24)$$

and identifying $\Lambda_{\mathrm{QCD}} = \frac{1}{z_0}$ and $Q = \frac{1}{z}$, we have at the cutoff $z = \epsilon = \frac{1}{\Lambda}$:

$$e^{\sqrt{\frac{2}{3}}\phi(\epsilon)} = \frac{11N_c}{6\pi}\alpha_{\mathrm{YM}}(\Lambda) = \frac{11N_c}{24\pi^2}g_{\mathrm{YM}}^2(\Lambda). \qquad (4.25)$$

Now, for a fluctuation $\varphi(z)$, we have

$$e^{\sqrt{2/3}(\phi(z)+\varphi(z)e^{iq\cdot x})} \approx e^{\sqrt{2/3}\phi(z)}\left(1 + \sqrt{2/3}\varphi(z)e^{iq\cdot x}\right). \qquad (4.26)$$

Now, $\varphi(z)$ near $z = 0$ behaves like any massless scalar fluctuation on an AdS background, and thus we have that it shifts the action by an amount

$$S_{5D} = \frac{1}{2\kappa^2}\varphi(\epsilon)^2 \frac{-1}{32}q^4 \log q^2/\mu^2 + \cdots. \tag{4.27}$$

The key now is to understand precisely which field theory correlator corresponds to taking the second derivative of this expression with respect to $\varphi(\epsilon)$. The field theory action is $-\frac{1}{4g_{\mathrm{YM}}^2}F^2$; effectively, we are adding a source by taking the coefficient to be instead $-\frac{1}{4g_{\mathrm{YM}}^2}(1 + \delta e^{iq\cdot x})$. Comparing this to eq. 4.26, we see that $\delta = -\sqrt{\frac{2}{3}}\varphi(\epsilon)$. Now, we have the two-point correlator for $G^2 = g_{\mathrm{YM}}^{-2}F^2$:

$$\int d^4x \left\langle \frac{1}{4}G^2(x)\frac{1}{4}G^2(0) \right\rangle e^{iqx} = -\frac{(N_c^2 - 1)}{64\pi^2}q^4 \log\frac{q^2}{\mu^2} + \ldots, \tag{4.28}$$

which should correspond to taking a second derivative with respect to δ of our above result, and so we find:

$$\frac{R^3}{\kappa^2} = \frac{64}{3}\frac{N_c^2 - 1}{64\pi^2} = \frac{(N_c^2 - 1)}{3\pi^2}. \tag{4.29}$$

In particular, for $N_c = 3$, this means that

$$S_{5D} = \frac{2}{3\pi^2 z_0^4}. \tag{4.30}$$

In order to find the actual condensate, we have to differentiate the action with respect to the value of the source on the boundary. In our case the source is just the QCD coupling itself g_{YM}^{-2}. Using the expression for the coupling in a pure YM theory

$$\alpha_{\mathrm{YM}}(Q) = \frac{2\pi}{\frac{11}{3}N_c \log\frac{Q}{\Lambda_{\mathrm{QCD}}}} \tag{4.31}$$

and identifying $\Lambda_{\mathrm{QCD}} = \frac{1}{z_0}$ we find that the derivative with respect to g_{YM}^{-2} (viewing g_{YM} as a function of z_0) is the same as $\frac{24\pi^2}{11N_c}z_0\frac{d}{dz_0}$.

Putting all this together, we find that:

$$\left\langle \frac{1}{4}\mathrm{Tr}F^2 \right\rangle = \frac{(N_c^2 - 1)}{12\pi^2}\frac{24\pi^2}{11N_c}4z_0^{-4} \approx (1.19z_0^{-1})^4. \tag{4.32}$$

Using our estimate of z_0 from the glueball mass in eq. 4.14, we obtain:

$$\left\langle \frac{1}{4\pi^2}\mathrm{Tr}F^2 \right\rangle = \frac{1}{\pi^2}(1.19 \times 680\text{ MeV})^4 \approx 0.043\text{ GeV}^4. \tag{4.33}$$

For comparison, an SU(3) lattice calculation found $\left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle \approx 0.10$ GeV^4 [26]. Our result is of the same order but slightly smaller. Most phenomenological estimates are smaller, beginning with the SVZ result of $\left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle \approx 0.012$ GeV^4, but for pure Yang-Mills the value is expected to increase [1].

4.3 Relation to analytic perturbation theory

The QCD perturbation series is an asymptotic expansion of some unknown function, and the divergence at Λ_{QCD} signals only a breakdown of perturbation theory, not a meaningful infinity. In particular, it has been proposed that the pole of the logarithm be cancelled by additional terms to produce an "analytic perturbation theory." See, for instance, the work of Shirkov and Solovtsov [27] and related literature (of which there is too much to give an exhaustive account here). It is interesting that our holographic equations produce a result along these lines, when interpreted in a particular way.

To see this, note that our identification of the coordinate z with the inverse of a renormalization group scale μ is only clearly defined in the far UV (near $z = 0$). In fact, when we take as a metric ansatz $\mathrm{d}s^2 = \exp(-2A(y))\mathrm{d}x^2 + \mathrm{d}y^2$, it is more reasonable to interpret $A(y)$ as $-\log \mu R$, so that the 4D part of the metric goes like $\mu^2 \mathrm{d}x^2$. Our expression for the QCD coupling in terms of the dilaton $\exp\sqrt{2/3}\phi(y)$ is given by $\frac{R}{y_0 - y}$, which blows up at a finite coordinate in exactly the way that the one-loop QCD beta function tells us α_s should blow up at $\mu = \Lambda_{\mathrm{QCD}}$. On the other hand, using the modified identification of the energy scale explained above,

$$A(y) \leftrightarrow -\log \mu R,$$

that is by identifying *the warp factor* (instead of y) as the logarithm of the energy scale, we find that $y \to y_0$ corresponds to $\mu \to 0$. Thus we can view $\exp\sqrt{2/3}\phi(y)$ as providing a formula for $\alpha_s(\mu)$ that is smoothly defined at all μ, which blows up as a power law in the deep infrared $\mu \to 0$ (instead of $\mu \to \Lambda_{\mathrm{QCD}}$) and reduces to the perturbative result at large μ.

In particular, one can solve for $\alpha_s(\mu)$ according to this prescription. The relevant expressions

$$\frac{6\pi}{11N_c}\alpha_s^{-1}(\mu) = (y_0 - y(\mu))/R \tag{4.34}$$

$$-\log \mu R = \frac{y(\mu)}{R} + \frac{1}{4}\log\frac{R}{y_0 - y(\mu)} \tag{4.35}$$

$$\Lambda_{\mathrm{QCD}} = z_0^{-1} = \frac{1}{R}e^{-y_0/R} \tag{4.36}$$

can be inverted to find

$$\frac{1}{\alpha_s(\mu)} = \frac{11N_c}{24\pi}W(4\mu^4/\Lambda_{\mathrm{QCD}}^4), \tag{4.37}$$

where $W(y)$ is the Lambert W-function [28], that is, the principal value of the solution to $y = x\exp(x)$. In fact, the Lambert W-function appears similarly in the analytic perturbation theory approach [29], although the form of $\alpha_s(\mu)$ is slightly different there. Nonetheless, our results are suggestive of a role for the backreaction on the metric as enforcing good analytic properties, which deserves further attention.

5. Effects of the $\mathrm{Tr}(F^3)$ condensate

In pure Yang-Mills, there is an operator of dimension 6, $\mathcal{O}_6 = f^{abc}F^a_{\mu\nu}F^b_{\nu\rho}F^c_{\rho\mu}$. This operator will get a condensate which modifies the OPE at higher orders. We wish to investigate the

size of the corrections on the glueball masses in our approach, to understand how stable the numerics are. Thus we add a new field χ, and we wish to modify the superpotential. We should still have in the potential terms Λ and $C \exp(b\phi)$, as before. However, now we should also have a mass term for χ, with $m_\chi^2 = 6(6-4) = 12$ by the AdS/CFT correspondence. On the other hand, our theory is not conformal, and $\mathcal{O}_6$ has an anomalous dimension proportional to α_s, suggesting we should also have terms in the potential that couple ϕ and χ.

For now, we will make no attempt to constrain all the higher-order terms in the 5D action coupling ϕ and χ. Instead, we seek a superpotential with the properties discussed above, as a first approximation. Luckily, there is a superpotential which allows the profiles of the scalars and the warp factor to be found analytically. Our motivation for this particular choice is that it allows an analytic solution and has the desired properties. On the other hand, it resembles certain superpotentials that arise in gauged supergravity [30]:

$$W(\phi, \chi) = \frac{1}{4} \exp\left(\sqrt{\frac{2}{3}}\phi\right) + \cosh(B\chi). \tag{5.1}$$

(We will see shortly that $B = 1$.) The corresponding potential is then:

$$
\begin{aligned}
V(\phi, \chi) &= 18\left[\left(\frac{\partial W}{\partial \phi}\right)^2 + \left(\frac{\partial W}{\partial \chi}\right)^2\right] - 12W^2 \\
&= -12 - 6e^{\sqrt{\frac{2}{3}}\phi} + (18B^4 - 12B^2)\chi^2 - 3B^2 e^{\sqrt{\frac{2}{3}}\phi}\chi^2 + \mathcal{O}(\chi^4). \tag{5.2}
\end{aligned}
$$

We find that $\phi(y)$ is as before, whereas $\chi(y)$ is given by $\chi'(y) = 6\frac{\partial W}{\partial \chi} = 6B \sinh(B\chi)$. But this is nearly identical to the equation we solved to find $\phi(y)$ in the case *without* log running. In particular, this means that

$$\chi(y) = \log\left(\tanh\frac{3(y_1 - y)}{R}\right), \tag{5.3}$$

where we have chosen $B = 1$ to ensure that at small z, $\chi(z) \sim z^6$. In fact we can check this, as in the potential above we expect $(18B^4 - 12B^2)\chi^2 = 6\chi^2$, confirming that we want $B = 1$.

Finally, we evaluate the warp factor, using $A'(y) = W(\phi(y), \chi(y))$:

$$A(y) = -\frac{1}{6}\log\left(\cosh\frac{3(y_1 - y)}{R}\sinh\frac{3(y_1 - y)}{R}\right) + \frac{1}{4}\log\left(\frac{R}{y_0 - y}\right) + \frac{y_1}{R} - \frac{\log 2}{3}, \tag{5.4}$$

where the first term replaces the y of our solution without the inclusion of $\mathcal{O}_6$, but deviates from it in the infrared. (The constant terms correct for a constant difference between y

and the first term, in the far UV.) The solution in the z coordinates is given by

$$ds^2 = \left(\frac{R}{z}\right)^2 \left[\left(\log \frac{z_0}{z}\right)^{\frac{1}{2}} \left(1 - \frac{z^{12}}{z_1^{12}}\right)^{\frac{1}{3}} dx^\mu dx^\nu \eta_{\mu\nu} - dz^2 \right] \tag{5.5}$$

$$\phi(z) = -\sqrt{\frac{3}{2}} \log \log \frac{z_0}{z} \tag{5.6}$$

$$\chi(z) = \log \frac{1 - \frac{z^6}{z_1^6}}{1 + \frac{z^6}{z_1^6}}. \tag{5.7}$$

At this point the reader should be notice certain issues in our calculation. First, while it is true that the potential $V(\phi, \chi)$ couples ϕ and χ and includes a term suggestive of an anomalous dimension, the solutions for $\phi(y)$ and $\chi(y)$ themselves are completely decoupled! This, however, is not really a concern: the backreaction on the metric feels all of the terms in the potential. In other words, it is *only through $A(y)$* that the anomalous dimension is manifest. If, as suggested earlier, we interpret $A(y)$ as $-\log \mu$, then $\phi(\mu)$ and $\chi(\mu)$ will feel the effect of the anomalous dimension.

Another issue is that we have sacrificed precise agreement with perturbation theory for the sake of having a simple, solvable example. We have arranged to get the logarithmic running of α_s, the proper scaling dimension of $\mathcal{O}_6$, and an anomalous dimension term for $\mathcal{O}_6$ which is proportional to α_s. On the other hand, we have not been careful to match the coefficient in this anomalous dimension. Details of the OPE and anomalous dimension for this operator can be found in ref. [31]; eventually one would want a model that matches them. Of course, in our discussion of $\alpha_s(\mu)$ earlier, we also had a second β function coefficient that did not match. This suggests that our analytically solvable superpotentials, while useful for a preliminary study, probably need to be replaced by a more detailed numerical study based on a more careful matching of the holographic renormalization group. Nonetheless, the disagreement appears only at higher orders of perturbation theory, and we can already use our preliminary superpotential $W(\phi, \chi)$ to get some sense of the stability of spectra calculated in holographic models.

5.1 Gubser's criterion: constraining z_1/z_0

In our solution, $\phi(z)$ blows up at $z = z_0$ while $\chi(z)$ blows up at $z = z_1$. The space will shut off at whichever of these is encountered first. Intuitively it is clear that if the dimension 6 condensate is to make a relatively small correction to the results we have already obtained using only the dimension 4 condensate, we should have $z_1 > z_0$, so that $\chi(z)$ remains finite over the interval where the solution is defined.

In fact there is a conjecture that will enforce this condition. Namely, Gubser in ref. [32] has proposed that curvature singularities of the type arising in the geometries we are considering are allowed only if the scalar potential is bounded above when evaluated on the solution. By "allowed", one should understand that in these cases one expects the singularity to be resolved in the full string theory; geometries violating the criterion are somehow pathological. The conjecture is based on some nontrivial consistency checks

involving considerations of finite temperature and examples from the Coulomb branch in AdS/CFT, but it is not proven. In any case we will assume for now that it holds for any geometry that can be properly thought of as dual to a field theory. It is clear that our original solution involving only ϕ satisfies the criterion: in that case we had $V(\phi) = -12 - 6e^{\sqrt{2/3}\phi} < 0$.

The case with χ is more subtle. We have

$$
\begin{aligned}
V(\phi(z), \chi(z)) &= -15 - 6e^{\sqrt{\frac{2}{3}}\phi(z)} \cosh \chi(z) + \cosh(2\chi(z)) \\
&= -6 \frac{\left(1 - \left(\frac{z}{z_1}\right)^{24}\right)}{\left(1 - \left(\frac{z}{z_1}\right)^{12}\right)^2 \log \frac{z_0}{z}} - 12 \frac{1 - 4\left(\frac{z}{z_1}\right)^{12} + \left(\frac{z}{z_1}\right)^{24}}{\left(1 - \left(\frac{z}{z_1}\right)^{12}\right)^2}
\end{aligned}
\tag{5.8}
$$

Clearly as $z_1 \to \infty$ we recover the previous solution and Gubser's criterion is satisfied. On the other hand, as soon as $z_1 < z_0$, V begins to attain large positive values as $z \to z_1$. The reason is simple: the function $1 - 4x^{12} + x^{24}$ has a zero at $x \approx 0.9$, so the second term above can attain positive values when $z \approx z_1$, and the denominator will attain arbitrarily small values provided the singularity at z_1 is reached (i.e. $z_1 < z_0$). In fact large positive values of V are attained if and only if $z_1 < z_0$.

In short, Gubser's criterion limits us to precisely those solutions which can be viewed in some sense as a perturbation of our existing solution.

5.2 Condensates

To calculate the condensate we need to again evaluate the classical action for the solution, which in our case is given by

$$
S = \frac{1}{2\kappa^2} \int_\epsilon^{z_0} \left(\frac{R}{z}\right)^6 \log \frac{z_0}{z} \left(1 - \frac{z^{12}}{z_1^{12}}\right)^{\frac{2}{3}} \left[-\frac{1}{2}(\phi'^2 + \chi'^2)\left(\frac{z}{R}\right)^2 \right.
\tag{5.9}
$$
$$
\left. + 12 + 6e^{\sqrt{\frac{2}{3}}\phi} \cosh \chi + 6 \sinh^2 \chi - \mathcal{R} \right].
$$

Again dropping the UV divergent terms we find either using (4.19) or by direct integration

$$
S = \frac{1}{2\kappa^2} \frac{(z_1^{12} - z_0^{12})^{\frac{2}{3}}}{2z_0^4 z_1^8}
\tag{5.10}
$$

Following the steps for calculating the condensate for the single field case we find that the modified condensate is given by

$$
\left\langle \frac{1}{4} \text{Tr} F^2 \right\rangle = \frac{(N_c^2 - 1)}{3\pi^2} \frac{24\pi^2}{11 N_c} z_0 \frac{d}{dz_0} \left[\frac{(z_1^{12} - z_0^{12})^{\frac{2}{3}}}{4z_0^4 z_1^8} \right] \approx (1.19 z_0^{-1})^4 \frac{1 + \left(\frac{z_0}{z_1}\right)^{12}}{\left[1 - \left(\frac{z_0}{z_1}\right)^{12}\right]^{\frac{1}{3}}}.
\tag{5.11}
$$

In the last step we have taken $N_c = 3$. One can see that for $z_1 > z_0$ (as expected from the criterion of section 5.1) this condensate is very insensitive to the actual value of z_1.

In order to actually fix z_1 one would have to calculate the second condensate $\langle \mathrm{Tr} F^3 \rangle$ and compare it to the lattice results. However, there are no reliable lattice estimates for this condensate available.

It is also plausible that if we account more properly for the anomalous dimension of $\mathrm{Tr} F^3$ and for matching of perturbative corrections to the OPE, we will be able to select a solution without this ambiguity. However, constructing such a solution appears to require a numerical study, which we will leave for future work.

5.3 Glueball spectra

Now that we have the background deformed by condensates of $\mathrm{Tr}\, F^2$ and $\mathrm{Tr}\, F^3$, we can again compute the glueball spectrum. What we would like to check is how sensitive the results are to the value of z_1. To simplify the numerical problem we are assuming that the low-lying glueball modes are still predominantly contained in the ϕ and A fields, and that the leading effect of turning on the χ field is to modify the gravitational background $A(y)$. Using this approximation we find the following equation satisfied by the glueball wave functions (again in z coordinates and units of R):

$$z^2 F''(z) - z \left(1 + \frac{5}{2 \log \frac{z_0}{z}} - \frac{4}{1 - \frac{z_1^{12}}{z^{12}}} \right) F'(z) + \tag{5.12}$$

$$\left(\frac{4}{\log \frac{z_0}{z}} \frac{1 + \frac{z^{12}}{z_1^{12}}}{1 - \frac{z^{12}}{z_1^{12}}} + \frac{m^2 z^2}{\left(1 - \frac{z^{12}}{z_1^{12}} \right)^{\frac{1}{3}} \sqrt{\log \frac{z_0}{z}}} - \frac{96 z^{12}}{z_1^{12} \left(1 - \frac{z^{12}}{z_1^{12}} \right)^2} \right) F(z) = 0.$$

One can see that the equation reduces to (4.13) in the limit when $z_1 \gg z_0 \geq z$. By again numerically solving this equation for various values of $z_1/z_0 > 1$ we find that the glueball eigenvalues are very insensitive to the actual value of z_1 as long as z_1 is not extremely close to z_0. For example, the lightest eigenvalue at $2.52/z_0$ increases by less than a percent while lowering z_1/z_0 from ∞ to 1.5. For $z_1/z_0 = 1.1$ the lightest mass grows by 3 percent, while for the extreme value of $z_1/z_0 = 1.01$ the growth is still just 9 percent. The glueball mass ratios are even less sensitive to z_1: the ratio of the first excited state to the lightest modes decreases by about 3 percent when changing z_1/z_0 from ∞ to 1.01. Thus we conclude that the predictions for glueball spectra presented in the previous section are quite robust against corrections from higher condensates.

6. Linearly confining backgrounds?

One of the most problematic aspects of holographic QCD is the deep IR physics: one expects Regge behavior from states of high angular momentum, and a linear confining potential. The solutions presented in this paper show many qualitative and quantitative similarities with ordinary QCD. However, they do not properly describe the highly excited glueball states. Since there is a singularity at a finite distance, the characteristic mass relation for highly excited glueballs will be that of ordinary KK theories (in this respect the theory is similar to the models with an IR brane put in by hand) $m_n^2 \sim n^2/z_0^2$, instead of

the expected Regge-type behavior $m_n^2 \sim n/z_0^2$ [8]. Experimental and lattice data suggest that linear confinement effects persist further into the UV than one might expect, and already the light resonances observed in QCD seem to fall on Regge trajectories. Regge physics arises naturally from strings; in our approach, the more massive excitations of the 5D string correspond to high-dimension operators on the field theory side. To describe linear confinement and Regge physics accurately, then, it is conceivable that one must take into account the effects of a large number of operators. Thus we are led to seek alternative, but still well-motivated, approaches to the deep IR physics. One approach is simply to demand that the 5D fields have IR profiles that provide the desired behavior, as in ref. [9]. However, one would like to have a dynamical model of this effect. Here we first check that the backgrounds used in sections 4-5 do not reproduce such IR profiles. However in [9] it was suggested suggested that tachyon condensation might provide the appropriate dynamics. We provide a simple modification of our model possibly substantiating this idea, and speculate on its relation to known gauge theory effects: namely, UV renormalons and other $1/Q^2$ corrections as discussed by Zakharov and others [35].

6.1 No linear confinement in the dilaton-graviton system

We first try to find other solutions that would not have a singularity at a finite distance, even for the action (4.1) since until now we have found only a particular solution for (4.1) using a superpotential. However in general there should be an infinite family of superpotentials giving the same potential. For the general case we cannot find the other solutions analytically for all values of the coordinates. We can, however, attempt to solve the equations of motion analytically close to the UV boundary. A similar approach has been taken in ref. [33] for a type 0 string theory containing a tachyon, considered to be dual to non-supersymmetric $SU(N)$ Yang-Mills with six adjoint scalars. In our case we will not consider a tachyon for now, as we consider 5D fields to be in one-to-one correspondence with gauge invariant operators in the 4D dual. There are some orientifold theories in type 0 that have no bulk tachyon, so our approach is not *a priori* meaningless.

In order to avoid the change of variables needed to achieve an asymptotically AdS metric (plus power corrections), we solve the equations beginning from the explicitly conformally flat parametrization of the metric,

$$\mathrm{d}s^2 = e^{-2A(z)} \left(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}z^2 \right). \tag{6.1}$$

Then for the Einstein equations and the equation of motion for ϕ we find (using $'$ to denote derivatives with respect to z):

$$\phi''(z) - 3A'(z)\phi'(z) - e^{-2A(z)}\partial V(\phi)/\partial\phi = 0, \tag{6.2}$$

$$6A'(z)^2 - (1/2)\phi'(z)^2 + e^{-2A(z)}V(\phi) = 0, \tag{6.3}$$

$$3A'(z)^2 - 3A''(z) + (1/2)\phi'(z)^2 + e^{-2A(z)}V(\phi) = 0. \tag{6.4}$$

We again use the same potential $V(\phi) = -6e^{\sqrt{2/3}\phi(z)} - 6$ (but no longer assume the simple

expression for the superpotential), and solve the equations in the UV to obtain

$$\phi(z) = \sqrt{\frac{3}{2}} \left(-\log\log\frac{1}{z} + \frac{3}{4}\frac{\log\log\frac{1}{z}}{\log\frac{1}{z}} + \frac{\gamma}{\log\frac{1}{z}} + \cdots \right), \qquad (6.5)$$

$$A(z) = \log\frac{1}{z} + \frac{1}{2\log\frac{1}{z}} - \frac{3}{8}\frac{\log\log\frac{1}{z}}{\left(\log\frac{1}{z}\right)^2} + \frac{\frac{\gamma}{2} + \frac{33}{48}}{\left(\log\frac{1}{z}\right)^2} + \cdots . \qquad (6.6)$$

The omitted terms are those which are smaller as $z \to 0$, i.e. power corrections in z or higher powers in $(-\log z)^{-1}$. Here γ is a parameter that is undetermined by the UV equations of motion, just as in the solution of ref. [33]. Further, we find that if we add small perturbations to $\exp(\sqrt{2/3}\phi)$ and to $A(z)$, an $\mathcal{O}(z^4)$ power correction to the former is allowed while only constant corrections to the latter are allowed. It is reassuring that we find a power correction suitable for the gluon condensate. Ref. [33] found also a correction of order z^2 and interpreted it as a UV renormalon[2] (whereas the power correction we have considered may be thought of as an IR renormalon). The tachyon was crucial to the appearance of the UV renormalon term, which might have interesting implications.

In order to find a linearly confining solution, we would need to connect these UV solutions to solutions in the $z \to \infty$ IR region, which would also give linear confinement. Suppose we then search for a solution that is valid at large z. In the case that there is no cosmological constant, there is a linear dilaton solution; this is clear since our equations are those for a noncritical string. The linear dilaton persists in the presence of a cosmological constant as a solution at large z; the equations are satisfied up to terms exponentially small at large z by a linear dilaton on flat 5D space. Such a linear dilaton background does *not* give a confining solution. Furthermore, one can check that no other power-law growth in z is allowed. In particular, the z^2 behavior as in ref. [9] is not a solution to our equations of motion. Thus we conclude that none of the solutions of the action (4.1) will result in a linearly confining background.

6.2 Linear confinement from the tachyon-dilaton-graviton system?

Although we do not understand how to *systematically* approach the study of UV renormalon corrections, it is at least superficially plausible that they are associated with a closed string tachyon. This is a natural scenario to consider, since the UV renormalon is associated with Q^{-2} corrections. Ref. [33] found such an association in a concrete string theory model. The study of such corrections has been discussed in great detail in the QCD literature, which links the idea of the UV renormalon to the quadratic corrections associated with the QCD string tension and with a hypothetical nonperturbative tachyonic gluon mass, as well as with monopoles and vortices; we can only refer to a small sample of that literature [35]. Holographically, effects associated with dimension 2 corrections would appear to be associated with tachyonic physics. Interestingly, it has been proposed that the UV renormalon is associated with a nonvanishing value of $\left\langle A^2 \right\rangle_{\text{min}}$ where the minimization is over gauge choices [36]. Such a dimension 2 condensate could plausibly be associated

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[2]For a review of the physics of the UV renormalon, see ref. [34].

holographically to a closed string tachyon (one that saturates the Breitenlohner-Freedman bound [37]), though it does contradict the picture of holographic fields as being associated to *gauge-invariant, local* operators in the field theory. The minimum value of the A^2 condensate can be expressed in terms of *nonlocal*, gauge-invariant operators, beginning with $F_{\mu\nu}(D^2)^{-1}F^{\mu\nu}$ [38, 39]. The nonlocality of this operator is perhaps suggestive of the long-distance, stringy effects of the flux tube needed to describe excited hadrons.

All of these hints of the importance of Q^{-2} corrections suggest that we take the idea of a closed string tachyon seriously, despite the lack of a gauge-invariant local operator for it to match to. Perhaps this indicates that certain 5D corrections can somehow be "resummed" into the effects of a tachyonic field. The linear confining potential of QCD has been suggested to relate to an $\exp cz^2$ background in 5D [9]. It has also been considered in the context of stringy physics and heavy quarks [40]. We would like to have a dynamical solution incorporating both asymptotic freedom in the UV and linear confinement in the IR. We have observed that the cosmological constant does not destroy the existence of a linear dilaton solution at large z (up to small corrections). This suggests that if we have a theory with a quadratic tachyon profile at zero cosmological constant, such a solution may persist in the presence of a cosmological constant.

The action of the bosonic noncritical string theory including the leading α' corrections, using a sigma model approach, is (after transforming to Einstein frame), according to [41]:

$$\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left[e^{\frac{4}{3}\Phi}\frac{4}{\alpha'}(\lambda-1)-2\mathcal{R}+\frac{8}{3}(\partial\Phi)^2+e^{-t}\left(\frac{4}{\alpha'}\left(1+t-\frac{1}{2}\lambda\right)e^{\frac{4}{3}\Phi}+\mathcal{R}\right.\right. \tag{6.7}$$

$$\left.\left.-\frac{4}{3}(\partial\Phi)^2-\frac{4}{3}\partial\Phi\partial t+(\partial t)^2\right)\right].$$

To this action we are adding a cosmological constant (which we assume could come from a 5-form flux) and adjust the parameters such that with the $t\to 0$ substitution we recover the action considered in the previous section. The resulting action is:

$$\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left[e^{\frac{4}{3}\Phi}\frac{12}{R^2}\frac{\lambda-1}{\lambda}+\frac{12}{R^2}-2\mathcal{R}+\frac{8}{3}(\partial\Phi)^2+\right. \tag{6.8}$$

$$\left.+e^{-t}\left(\frac{12}{R^2\lambda}\left(1+t-\frac{1}{2}\lambda\right)e^{\frac{4}{3}\Phi}+\mathcal{R}-\frac{4}{3}(\partial\Phi)^2-\frac{4}{3}\partial\Phi\partial t+(\partial t)^2\right)\right].$$

The resulting equations of motion for the metric ansatz $ds^2=e^{2A(z)}(dx^2-dz^2)$ are:

$$\frac{4}{3}e^{2A}e^{\frac{4}{3}\Phi}\frac{12}{R^2\lambda}(\lambda-1)+\frac{16}{3}(3A'\Phi'+\Phi'') \tag{6.9}$$

$$+e^{-t}\left(e^{\frac{4}{3}\Phi+2A}\frac{12}{R^2}\frac{1}{\lambda}\left(1+t-\frac{1}{2}\lambda\right)\frac{4}{3}+\frac{8}{3}t'\Phi'-8A'\Phi'-\frac{8}{3}\Phi''+\frac{4}{3}t'^2-4A't'\right)=0$$

$$\frac{12}{R^2\lambda}e^{\frac{4}{3}\Phi+2A}\left(\frac{\lambda}{2}-t\right)-12A'^2-8A''-\frac{4}{3}\Phi'^2-t'^2-4A'\Phi'-\frac{4}{3}\Phi''+2t''+6A't'=0$$

$$\tag{6.10}$$

$$-6A'^2(2-e^{-t})+\frac{6}{R^2}e^{2A}\left(1+\frac{1}{\lambda}e^{\frac{4}{3}\Phi}\left(\lambda-1+e^{-t}\left(1-\frac{\lambda}{2}+t\right)\right)\right) \tag{6.11}$$

$$+\Phi'^2\left(\frac{2}{3}-\frac{4}{3}e^{-t}\right)+\frac{1}{2}e^{-t}\left(-\frac{4}{3}\Phi't'+t'^2\right)=0.$$

Here $\lambda = (26 - D)/3 = 21/3$. One can show that for large z the leading order solution to these equations is given by

$$\Phi \to \Phi_0, \quad A \to A_0, \quad t \to -\frac{3}{\lambda} e^{2A_0 + \frac{4}{3}\Phi_0} z^2 + \frac{1}{2}(\lambda - 2). \qquad (6.12)$$

In order to serve our purposes there must be a solution interpolating between logarithmic running on AdS in the UV and the above confining background (with flat space and constant dilaton) in the IR. It would be interesting to numerically study these equations, as well as similar systems in the type 0 string. The IR solution above has the property that the e^{-t} terms are *growing* at large z, whereas the proposed background in ref. [9] has an exponential *shutoff* of the metric, which is not a solution of the above equations. However, the sign does not appear to affect the existence of Regge physics. Regardless, we intend these remarks not as a definitive statement, but as a provocative hint that further studies of tachyon dynamics could be useful for understanding QCD.

7. Conclusions and outlook

In this paper we have clarified a number of aspects of AdS/QCD, and sketched a concrete program for computing in the holographic model and estimating associated errors. We have also explained that deep IR physics, associated with high radial excitations or large angular momentum, is troublesome in this framework. The underlying reason is that the OPE does not reproduce the quadratic corrections associated with this physics. We have suggested that a closed string tachyon can reproduce much of the underlying dynamics, but at this point that is a toy model and it is not clear how to systematically apply the idea to computations.

Our results suggest a number of directions for new work. One obvious task is to extend these results to theories with flavor. This should be straightforward, although there are potential numerical difficulties. It would be particularly interesting to see if one can obtain results for mixing of glueballs with $q\bar{q}$ mesons without large associated uncertainties.

Another direction is to make more explicit the connection between the renormalization group and the holographic dual. The 5D action has an interpretation as the 4D generating functional $W[J]$, whereas some numerical attempts involving truncations of exact RGEs have focused on computing the Legendre transform of this quantity, $\Gamma[\phi]$. There are subtle issues of nonperturbative gauge-invariant regulators that must be considered in such studies, but it is conceivable that such existing work could be reinterpreted as a computation of background profiles for various fields, about which we could then compute the spectrum of excitations and couplings with the usual 5D techniques. We hope to clarify this relationship in a future paper. It is also interesting in this context to think about how the holographic renormalization group might relate to the "analytic perturbation theory" framework.

Finally, the intricate story relating $1/q^2$ corrections, UV renormalons, vortices, linear confinement, and the closed string tachyon is very interesting and still rather poorly understood. A better understanding of these quantities and their relationships could elucidate

the confining dynamics of QCD, and also shed light on closed string tachyon condensation in general (with possible applications to cosmology).

Acknowledgments

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